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FI MU Report Series FIMU-RS-99-06

 September 1999

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Abstract. Report discusses contribution of Birkhoff's aesthetic measure to formal aesthetic evaluation of regular geometrical objects, namely Chinese vases. Characteristics of aesthetic measure function are considered in order to verify correctness of such application and extended Birkhoff's aesthetic measure is then introduced.

Keywords : exact (numerical) aesthetics, Birkhoff's aesthetic measure, extended Birkhoff's aesthetic measure of vases

1 Introduction

Art and manufacture of aesthetic criteria fulfilling artifacts accompany the humankind from time immemorial. Pristine ritual nature of an artistic expression evolved into premeditated production and admiration of aesthetic, sensually engaging artifacts that have endured in a form of material or immaterial artistic creations up to the present days. For each historical period and prevailing artistic movement we can observe certain binding rules for assessing "beauty" or "artistic value". These rules have varied (often startlingly) from decade to decade; qualities praised by daedal Art Nouveau, for instance, are speedily rejected by pragmatic Functionalism.

One can ask whether it is possible to design a formal aesthetic evaluation apparatus for a certain class of objects, valid without regard to artistic style paradigms. Is there anything such as an "absolute art measure" ? Can art be appraised on a set of accurate rules ? Initial answer to this problem was given in the first half of the twentieth century by a mathematician George D. Birkhoff, who proposed his aesthetic measure theory [1] binding the aesthetic perception with two criteria : order and complexity.

In case of visual arts, *order* O depends on geometrical relations among identifiable segments of an evaluated object (e.g. curves or planes). Attributes such as symmetry and balance are considered to be relevant for intense aesthetic perception.

Complexity C is "the number of localities our sight will spontaneously rest on". According to Birkhoff, complexity negatively effects overall aesthetic measure since complex objects tend to deflect onlooker's contemplation.

Birkhoff's aesthetic measure M is then defined as a division of order and complexity[1] :

$$M = O/C \tag{1}$$

In order to evaluate a certain class of objects an examination of principles for order and complexity definitions would follow.

2 Birkhoff's aesthetic measure of vases

For the aesthetic measure research a class of ancient Chinese vases was chosen. These vases are exceptionally suitable for such purpose since Chinese master craftsmen (vase manufacture was treated as a separate branch of art) particularly put an emphasis on pulchritude expression by geometrical means with minimum decorative "surplus" elements protruding vase contour. It is this contour, or a silhouette, that is going to be evaluated; it is generally easy to capture and we humans have a lot of perceptual experience in percieving silhouettes. Vase decor contribution to the aesthetic experience will not be considered for this moment. Geometrical representation of a Chinese vase together with algorithms for order and complexity computation were introduced by Ewa J. Grabska et al. in [2].

The examined class of vases consists of objects created by revolving a *characteristic curve* in a certain distance around vertical axis. Each vase can be thus projected to its planar *outline*, e.g. a pair of symmetrical characteristic curves connected by two elliptical curves at neck and foot.

Along the vase outline several types of *characteristic points* that belong to characteristic curves can occur :

- Terminal points.
- Inflex points.
- Points of vertical tangent.
- Points of steep change of tangent direction.

Tangents at the characteristic points are called *characteristic tangents*.

Vase *centers* are located on the rotation axis at points of minimal and maximal distances from the characteristic curve.

The rotation axis, parallel lines passing through the characteristic points and their perpendiculars together with characteristic tangents at these points define a *characteristic network* which is put down as a geometrical configuration of the evaluated vase.

To apply formula (1) it is necessary to express order and complexity of the vase being evaluated by means of configuration parameters mentioned above.

Let *horizontal order* H be the number of independent relations of ratios 1 : 1 and 2 : 1 within pairs of horizontal distances h_i, h_j between symmetrical characteristic points, $H \leq 4$.

[1] Therefore, Birkhoff's proposition can be correctly applied on static, regular artifacts that respect proportional canons and can be described in a language of geometry with curves or planes. For dynamic creations such approach will be inadequate.

Let *vertical order* V be the number of independent relations of ratios $1:1$ and $2:1$ within pairs of adjacent vertical distances v_i, v_j between characteristic points, $V \leq 4$.

Let *proportional order* P be the number of independent relations of ratios $1:1$ and $2:1$ within pairs of horizontal and adjacent vertical distances h_i, v_j between characteristic points, $P \leq 2$.

Let *tangent order* T be the number of following independent relations $\mathcal{T}$, $T \leq 4$:

– Perpendicularity of characteristic tangents.
– Parallelism of nonvertical characteristic tangents.
– Verticality of a characteristic tangent at the terminal or inflex points.
– Intersection of a characteristic tangent or its normal with the vase center.

Let complexity be the number of characteristic points, typically $C \in \langle 6; 20 \rangle$.

Birkhoff's aesthetic measure is defined as a sum of partial orders divided by complexity:

$$M = \frac{H + V + P + T}{C} \tag{2}$$

3 Birkhoff's aesthetic measure characteristics

Formula (2) puts in conjunction the aesthetic measure with a geometrical shape. It can be positively applied on vase design process where the aesthetic measure value and certain proportional requirements are given. However, it is not evident whether this apparatus corresponds with the converse process — its application on a particular vase for the aim of its aesthetic evaluation. To verify this hypothesis we are interested in the aesthetic measure behaviour in case of gradual vase outline distortion.

For the aesthetic measure research two items were selected from a set of Chinese vase photographs[2] with dissimilar proportions and a significant difference between their aesthetic measure values. Outlines and characteristic networks of the chosen vases are shown in Fig. 1.

These vases constitute border elements of a sequence $(v_0, \ldots, v_{14})$ morphing vase v_0 onto v_{14}. Thirteen experimental transitional vases were generated using MDMorph program (see [3]); central part of the sequence can be seen in Fig. 2. Consecutively, formula (2) was applied on each vase to evaluate its aesthetic measure. The obtained results are overviewed in Table 1.

From the graph in Fig. 3 it is obvious that within the examined domain of fifteen vases the aesthetic measure function gains only five values, with two the most frequent ones (represented by six vases identically) being also the lowest (1/12 and 2/12). The characteristic network is guided by the number of symmetrical characteristic point pairs, which demarcate individual characteristic curve segments. Shape distortion of any segment consequently results in global

[2] Photographs of Chinese vases have been taken with a kind permission of Dr. Alena Křížová in the Museum of Applied Arts depository, Moravian Gallery in Brno.

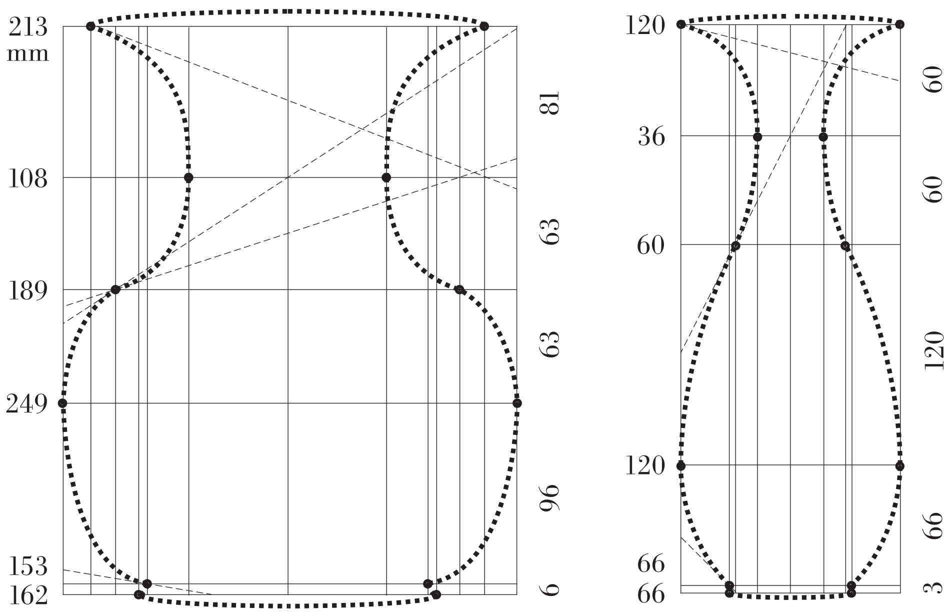

Fig. 1. Outlines and characteristic networks of vases v_0 and v_{14}.

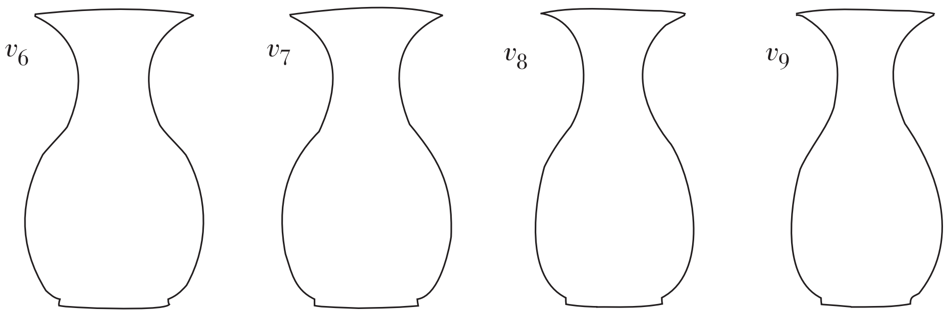

Fig. 2. Gradual shape distortion between experimental vases v_6 and v_9.

Table 1. Aesthetic measure M applied on shape distortion sequence.

	v_0	v_1	v_2	v_3	v_4
M	$2/12$	$1/12$	$1/12$	$2/12$	$1/12$
	v_5	v_6	v_7	v_8	v_9
M	$1/12$	$2/12$	$5/12$	$1/12$	$1/12$
	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}
M	$2/12$	$2/12$	$2/12$	$3/12$	$10/12$

ratio variation for distances and angle variation for tangents in the characteristic network. In case of neighbouring vases v_7 and v_8, whose outlines differ with the maximum deflection equal to $3/100$ of their height, the aesthetic measure difference is $4/12$; in case of vases v_{12} and v_{14} and deflection $6/100$ of the height the difference is $8/12$ already, provided that the greatest aesthetic measure difference of the whole sequence equals $9/12$. By a gradual vase outline distortion the aesthetic measure increment is not smooth and the function shows noncontinuous characteristic.

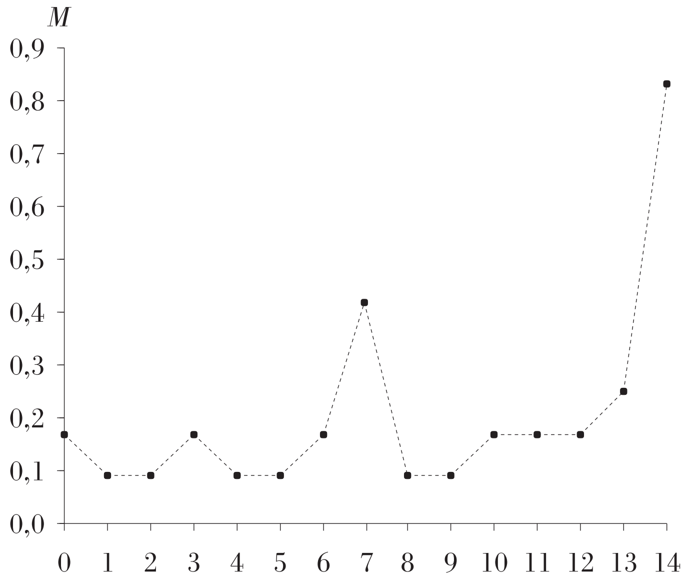

Fig. 3. Graph of Birkhoff's aesthetic measure M.

4 Extended Birkhoff's aesthetic measure

Noncontinuous characteristic of the aesthetic measure function is expectable owing to the unitary order increment with a relatively low upper bound; such an increment can disproportionately influence the considered function. Therefore, *length tolerance* ε for characteristic network distance ratios and *angle tolerance* ζ for characteristic tangents are worth considering.

Let us keep the unitary increment in case of perfect distance ratios $l_1/l_2 = k$, $k \in \{\frac{1}{2}, 1, 2\}$, and characteristic tangent (or its normal) relations $\mathcal{T}$ of perpendicularity, parallelism, verticality and vase center crossing.

Let us introduce new increment values in case of distance ratio deflections $k\,\frac{l_1}{l_2} - 1 \in (0,\varepsilon\rangle$, $k \in \{\frac{1}{2}, 1, 2\}$, and length tolerance ε :

$$R\,(l_1, l_2) = \begin{cases} \max(0; 1 - 10\,(k\,\frac{l_1}{l_2} - 1)) & k\frac{l_1}{l_2} - 1 \in \langle 0;\varepsilon\rangle \\ 0 & \text{otherwise} \end{cases}$$

Let us introduce new increment values also in case of characteristic tangent (or its normal) deflections $\eta \leq \zeta$ from relations $\mathcal{T}$ for noncontiguous tangents and angle tolerance ζ; by contiguousness we consider the property of tangents passing through a certain characteristic point and sharing an angle $(0;\zeta\rangle$:

$$Q\,(t_1, t_2) = \begin{cases} \max(0; 1 - \eta/10) & t_1 \pm \eta, t_2 \pm \eta \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

Let extended partial orders with length and angle tolerances ε, ζ be formally defined as follows :

$$H(\varepsilon) = \sum_{\substack{i,j=1..\frac{C}{2},\\ i<j}} R\,(h_i, h_j)$$

$$V(\varepsilon) = \sum_{\substack{i=1..\frac{C}{2}-1,\\ j=1..i-1,\\ k=i..\frac{C}{2}-1}} R\left(\sum_{m=j}^{i-1} v_m, \sum_{n=i}^{k} v_n\right)$$

$$P(\varepsilon) = \sum_{\substack{i=1..\frac{C}{2},\\ j=1..i-1,\\ k=i..\frac{C}{2}-1}} R\left(h_i, \sum_{m=j}^{i-1} v_m\right) + R\left(h_i, \sum_{n=i}^{k} v_n\right)$$

$$T(\zeta) = \sum_{\substack{t_i,t_j\\ \text{tangents}}} Q\,(t_i, t_j)$$

Upper bound values are the same as in case of partial orders without tolerances.

Extended Birkhoff's aesthetic measure is defined as a sum of extended partial orders divided by complexity :

$$M(\varepsilon,\zeta) = \frac{H(\varepsilon) + V(\varepsilon) + P(\varepsilon) + T(\zeta)}{C} \tag{3}$$

Finally, we have to choose proper tolerances ε and ζ. Let us set the length tolerance $\varepsilon = 0.1$ and the angle tolerance $\zeta = 5°$. If formula (3) together with considered tolerances is applied on the vase sequence, results overviewed in Table 2 are obtained.

The extended aesthetic measure function $M(\varepsilon,\zeta)$ tends to respect a progress of function M in each of its monotonous intervals, as demonstrated in Fig. 4. This time, adequate distribution of its values was attained. The greatest difference of the aesthetic measure among neighbouring vases equals $3.1/12$ (vases

Table 2. Extended aesthetic measure $M(\varepsilon, \zeta)$ applied on shape distortion sequence.

	v_0	v_1	v_2	v_3	v_4
$M(\varepsilon, \zeta)$	4.3/12	3.9/12	4.0/12	4.0/12	3.0/12
	v_5	v_6	v_7	v_8	v_9
$M(\varepsilon, \zeta)$	2.8/12	4.5/12	5.9/12	5.3/12	4.3/12
	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}
$M(\varepsilon, \zeta)$	4.9/12	4.9/12	5.6/12	6.9/12	10.0/12

v_{13} and v_{14}); the remaining neighbouring vases do not show differences higher than 1.7/12. These facts attest that the extended aesthetic measure characterisctic can be considered now as continuous.

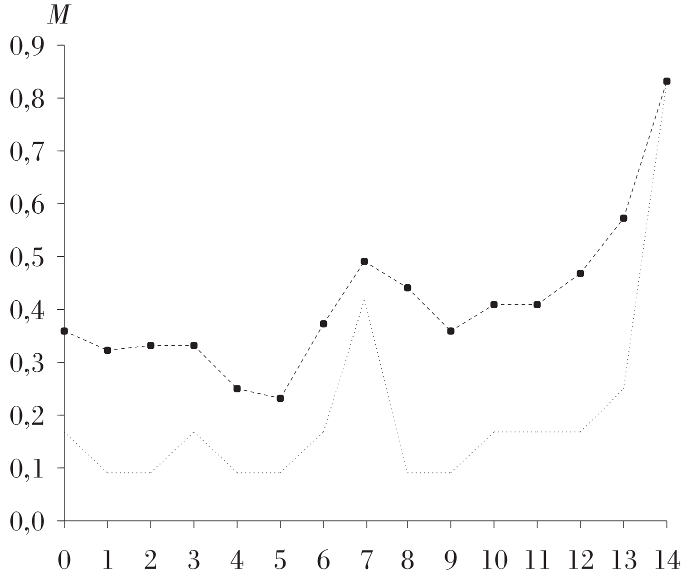

Fig. 4. Graph of extended Birkhoff's aesthetic measure $M(\varepsilon, \zeta)$.

5 Conclusion

This aesthetic measure study sequels an earlier research [2] by Grabska et al., where formula (2) for the aesthetic measure evaluation was introduced together with its generative appliance on experimental vases with a given aesthetic measure value. The aim of this paper was to verify whether Birkhoff's aesthetic measure would be applicable in the converse process, e.g. for correct aesthetic evaluation of considered class members.

The examined feature was a characteristic of the aesthetic measure function in case of gradual vase shape distortion. If formula (2) is applied on a sequence of distorted vases, smooth aesthetic measure increments cannot be expected. Vases with similar outlines may vary significantly in computed values. Since slight inaccuracies in ancient vase shape are expectable (they could also appear during photograph taking or processing), this formula is correctly applicable in (computer-aided) vase design only; in that case manufactured items that presumably differ from a template vase are not supposed to be evaluated retrospectively.

Major improvement in the aesthetic measure function progress was achieved by extension (3) of Birkhoff's measure with a refinement of partial order increment values. This function, tolerating deflections of characteristic network relations, already appears to be convenient for a formal vase aesthetic evaluation. Specifically, the length tolerance $\varepsilon = 0.1$ and the angle tolerance $\zeta = 5^\circ$ were considered.

Further steps in Birkhoff's aesthetic measure research should incorporate more detailed analysis of conditions that influence order and complexity values; this process may unveil new partial order definitions. Birkhoff's extended aesthetic measure formula (3) might also be redefined in a way that associates partial orders with weights according to a rate of their representation in the aesthetic perception. Replacing the sum of partial orders in order O definition with some more composite function may not be precluded. Likewise, to enrich the aesthetic measure with nonrational compositional ratios, such as a golden section, is worth considering.

Whether these suggestions will meet the conception of both computer scientists engaging in exact aesthetics and art theoreticians in order to find a proper formal aesthetic evaluation tool is the matter of subsequent research.

References

1. George D. Birkhoff. *Aesthetic Measure.* Cambridge Massachusetts' University Press, 1933.
2. Sabina Denkowska, Ewa J. Grabska, Katarzyna Marek. Application of Birkhoff's aesthetic measure to computer aided design of vases. *Machine GRAPHICS & VISION*, 7(1/2):69–75, 1994.
3. Martin Dobšík. Shape warping by grid deformation (in Czech). Master's thesis, Technical University of Brno, FEI, Brno, 1997.

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