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Abstract

We compare the classes of behaviours (transition systems) which can be generated by normed BPA_τ and normed BPP_τ processes. We exactly classify the intersection of these two classes, i.e. the class of transition systems which can be equivalently (up to bisimilarity) described by the syntax of normed BPA_τ and normed BPP_τ processes. We provide such a characterisation for classes of normed BPA and normed BPP processes as well.

Next we show that it is decidable in polynomial time whether for a given normed BPA_τ (or BPP_τ respectively) process Δ there is some (unspecified) normed BPP_τ (or BPA_τ respectively) process Δ' such that Δ is bisimilar to Δ'. Moreover, if the answer is positive then our algorithms also construct the process Δ'. Simplified versions of the algorithms mentioned above for normed BPA and normed BPP are given too.

As an immediate (but important) consequence we also obtain the decidability of bisimilarity in the union of normed BPA_τ and normed BPP_τ processes.

1 Introduction

We study a relationship between the classes of transition systems, which are generated by normed BPA_τ [BK88] and normed BPP_τ [Chr93] processes. We also examine such a relationship between their respective subclasses, namely normed BPA and normed BPP processes. BPA processes form type 2 class of prefix processes in Chomsky hierarchy for processes as given in

[Sti96] and [Mol96], whereas BPP form type 2 class of commutative processes in this hierarchy.

BPA processes can be seen as simple sequential programs (they are equipped with a binary sequential operator). This class of processes has been intensively studied by many researchers. Baeten, Bergstra and Klop proved in [BBK87] that bisimilarity is decidable for normed BPA processes. Much simpler proofs of this were later given in [Cau88], [HS91] and [Gro91]. In [HS91] Hüttel and Stirling used a tableau decision method and gave also sound and complete equational theory. Hirshfeld, Jerrum and Moller demonstrated in [HJM94a] that the problem is decidable in polynomial time. The decidability result was later extended to the whole class of BPA processes by Christensen, Hüttel and Stirling in [CHS92].

If we replace the binary sequential operator with the parallel operator, we obtain BPP processes. They can thus be seen as simple parallel programs. Christensen, Hirshfeld and Moller proved in [CHM93] that bisimilarity is decidable for BPP processes. A polynomial decision algorithm for normed BPP processes was presented in [HJM94b] by Hirshfeld, Jerrum and Moller.

If we allow a parallel operator not to specify just merge but also an internal communication between two BPP processes resulting in a special action τ, we obtain the class of BPP_τ processes ([Chr93]). In order to compare this class with its sequential counterpart we employ the class of BPA_τ processes ([BK88]). Decidability and complexity results just mentioned hold for these classes as well.

An interesting problem is, what is the exact relationships between BPA_τ and BPP_τ processes and that between their subclasses BPA and BPP, i.e. what is the relationship between sequencing and parallelism (possibly allowing simple communication). We answer these questions for normed subclasses of processes just mentioned. In the sequel we denote these subclasses by nBPA_τ, nBPA, nBPP_τ and nBPP respectively.

Our paper is organised as follows. First we recall some basic definitions and properties of BPA_τ, BPP_τ and regular processes which are relevant to the subject of our paper. In Section 3 we give an exact characterisation of those transition systems which can be equivalently (up to bisimilarity) described by the syntax of nBPP_τ and nBPA_τ processes. Next we show that if we restrict ourselves to nBPA and nBPP processes we obtain much simpler (and hopefully nice) characterisation of those behaviours which are common to these subclasses. In Section 4 we demonstrate it is decidable whether for a given nBPA (nBPA_τ, nBPP_τ, nBPP respectively) process Δ there is some (unspecified) nBPP (nBPP_τ, nBPA_τ, nBPA respectively) process Δ' such that $\Delta \sim \Delta'$. These algorithms are polynomial. We also show that if

the answer to the previous question is positive, then the process Δ' can be effectively constructed. Hence, as an important consequence we also obtain the decidability of bisimulation equivalence in the union of normed BPA_τ and normed BPP_τ processes. We conclude with remarks to related works and future research.

In many constructions of our paper we use the fact that regularity is decidable for normed BPA and normed BPP processes in polynomial time (a process is regular if it is bisimilar to a process with finitely many states). Regularity of BPA processes was examined for the first time by Mauw and Mulder in [MM94], but their notion of regularity is different from the usual one. Kučera showed in [Kuč95] that the result of Mauw and Mulder can be used to decide regularity of normed BPA processes (BPA_τ case is an easy consequence) and that regularity of normed BPP and normed BPP_τ processes is also decidable. These algorithms are polynomial.

2 Basic definitions, preliminary knowledge

2.1 BPA_τ and BPP_τ processes

Let $\Lambda = \{a, b, c, \ldots\}$ be a countably infinite set of *atomic actions* such that for each $a \in \Lambda$ there is a corresponding *dual* action $\overline{a}$ with the convention that $\overline{\overline{a}} = a$. Let $Act = \Lambda \cup \{\tau\}$ where $\tau \notin \Lambda$ is a special (silent) action. Let $Var = \{X, Y, Z, \ldots\}$ be a countably infinite set of *variables* such that $Var \cap Act = \emptyset$. The classes of recursive BPA_τ and BPP_τ expressions are defined by the following abstract syntax equations:

$$\begin{array}{lcl} E_{BPA_\tau} & ::= & a \mid X \mid E_{BPA_\tau}.E_{BPA_\tau} \mid E_{BPA_\tau} + E_{BPA_\tau} \\ E_{BPP_\tau} & ::= & a \mid X \mid aE_{BPP_\tau} \mid E_{BPP_\tau}|E_{BPP_\tau} \mid E_{BPP_\tau} + E_{BPP_\tau} \end{array}$$

Here a ranges over Act (note that we allow also τ as an action prefix) and X ranges over Var. The symbol Act^* denotes the set of all finite strings over Act.

As usual, we restrict our attention to guarded expressions. A BPA_τ or BPP_τ expression E is *guarded* if every variable occurrence in E is within the scope of an atomic action.

A *guarded BPA_τ (or BPP_τ) process* is defined by a finite family Δ of recursive process equations

$$\Delta = \{X_i \stackrel{def}{=} E_i \quad | \quad 1 \leq i \leq n\}$$

where X_i are distinct elements of Var and E_i are guarded BPA_τ (or BPP_τ) expressions, containing variables from $\{X_1, \ldots, X_n\}$. The set of variables which appear in Δ is denoted by $Var(\Delta)$.

The variable X_1 plays a special role (X_1 is sometimes called *the leading variable*—it is a root of a labelled transition system, defined by the process Δ and following rules:

$$\frac{}{a \stackrel{a}{\to} \epsilon} \qquad \frac{E \stackrel{a}{\to} E'}{E.F \stackrel{a}{\to} E'.F} \qquad \frac{E \stackrel{a}{\to} E'}{E + F \stackrel{a}{\to} E'} \qquad \frac{F \stackrel{a}{\to} F'}{E + F \stackrel{a}{\to} F'} \qquad \frac{E \stackrel{a}{\to} E'}{E|F \stackrel{a}{\to} E'|F}$$

$$\frac{F \stackrel{a}{\to} F'}{E|F \stackrel{a}{\to} E|F'} \qquad \frac{E \stackrel{a}{\to} E' \quad F \stackrel{\overline{a}}{\to} F'}{E|F \stackrel{\tau}{\to} E'|F'}\ (a \neq \tau) \qquad \frac{E \stackrel{a}{\to} E'}{X \stackrel{a}{\to} E'}\ (X \stackrel{def}{=} E \in \Delta)$$

The symbol ϵ denotes the empty expression with usual conventions: $\epsilon|E = E$, $E|\epsilon = E$ and $\epsilon.E = E$. Nodes of the transition system generated by Δ are BPA_τ (or BPP_τ) expressions, which are often called *states of* Δ, or just "states" when Δ is understood from the context. We also define the relation $\stackrel{w}{\to}^*$, where $w \in Act^*$, as the reflexive and transitive closure of $\stackrel{a}{\to}$ (we often write $E \to^* F$ instead of $E \stackrel{w}{\to}^* F$ if w is irrelevant). Given two states E, F, we say that F is *reachable from* E, if $E \to^* F$. States of Δ which are reachable from X_1 are said to be *reachable*.

Remark 1. *Processes are often identified with their leading variables. Furthermore, if we assume a fixed process Δ, we can view any process expression E (not necessarily guarded) whose variables are defined in Δ as a process too; we simply add a new leading equation $X \stackrel{def}{=} E'$ to Δ, where X is a variable from Var such that $X \notin Var(\Delta)$ and E' is a process expression which is obtained from E by substituting each variable in E with the right-hand side of its corresponding defining equation in Δ (E' must be guarded now). All notions originally defined for processes can be used for process expressions in this sense too.*

2.1.1 Bisimulation

The equivalence between process expressions (states) we are interested in here is *bisimilarity* [Par81], defined as follows:

Definition 1. *A binary relation R over process expressions is a* bisimulation *if whenever $(E, F) \in R$ then for each $a \in Act$*

- *if $E \stackrel{a}{\to} E'$, then $F \stackrel{a}{\to} F'$ for some F' such that $(E', F') \in R$*
- *if $F \stackrel{a}{\to} F'$, then $E \stackrel{a}{\to} E'$ for some E' such that $(E', F') \in R$*

Processes Δ *and* Δ' *are* bisimilar, *written* $\Delta \sim \Delta'$, *if their leading variables are related by some bisimulation.*

2.1.2 Normed processes

An important subclass of BPA_τ and BPP_τ processes can be obtained by an extra restriction of *normedness*. A variable $X \in Var(\Delta)$ is *normed* if there is $w \in Act^*$ such that $X \stackrel{w}{\rightarrow}{}^* \epsilon$. In that case we define the *norm* of X, written $|X|$, to be the length of the shortest such w. In case of BPP_τ processes we also require that no τ action which appears in w is a result of a communication on dual actions in the sense of operational semantics given above. This is necessary if we want the norm to be additive over '|' operator (τ can still occur in w—remember it can be used as an action prefix). A process Δ is *normed* if all variables of $Var(\Delta)$ are normed. The norm of Δ is then defined to be the norm of X_1. Note the norm of a normed process is easy to compute and bisimilar processes must have the same norm.

2.1.3 Greibach normal form

Any BPA_τ or BPP_τ process Δ can be effectively presented in so-called 3-Greibach normal form (see [BBK87] and [Chr93]). Before the definition we need to introduce the set $Var(\Delta)^*$ of all finite sequences of variables from $Var(\Delta)$, and the set $Var(\Delta)^\otimes$ of all finite multisets over $Var(\Delta)$. Each multiset of $Var(\Delta)^\otimes$ denotes a BPP_τ expression by combining its elements in parallel using the '|' operator.

Definition 2. *A* BPA_τ *(or* BPP_τ*) process* Δ *is said to be in* Greibach normal form (GNF) *if all its equations are of the form*

$$X \stackrel{def}{=} \sum_{j=1}^{n} a_j \alpha_j$$

where $n \in N$, $a_j \in Act$ *and* $\alpha_j \in Var(\Delta)^*$ *(or* $\alpha_j \in Var(\Delta)^\otimes$*). If* $Length(\alpha_j) \leq 2$ *(or* $card(\alpha_j) \leq 2$*) for each* j, $1 \leq j \leq n$, *then* Δ *is said to be in 3-GNF.*

From now on we assume that all BPA_τ and BPP_τ processes we are working with are presented in GNF. This justifies also the assumption that all reachable states of a BPA_τ process Δ are elements of $Var(\Delta)^*$ and all reachable states of a BPP_τ process Δ' are elements of $Var(\Delta')^\otimes$.

Remark 2. *In the rest of this paper we let Greek letters $\alpha, \beta, \ldots$ range over reachable states of a BPA_τ or BPP_τ process Δ in GNF. Occasionally we also use the notation α^i with the following meaning:*

$$\alpha^i = \underbrace{\alpha.\alpha \cdots \alpha}_{i} \quad \textit{if } \alpha \textit{ is a state of some } BPA_\tau \textit{ process in GNF}$$

$$\alpha^i = \underbrace{\alpha|\alpha \cdots |\alpha}_{i} \quad \textit{if } \alpha \textit{ is a state of some } BPP_\tau \textit{ process in GNF}$$

2.2 Regular processes

Many proofs in this paper take advantage of the fact that regularity of normed BPA_τ and normed BPP_τ processes is decidable in polynomial time. Regularity of BPA processes was examined for the first time by Mauw and Mulder in [MM94], but their notion of regularity is different from the usual one. Kučera showed in [Kuč95] that the result of Mauw and Mulder can be used to decide regularity of normed BPA (and thus also BPA_τ) processes and that regularity of normed BPP and BPP_τ processes is also decidable. These algorithms are polynomial. The next definition explains what is meant by the notion of regularity and introduce standard normal form for regular processes.

Definition 3. *A process Δ is* regular *if there is a process Δ' with finitely many states such that $\Delta \sim \Delta'$. A regular process Δ is said to be in normal form if all its equations are of the form*

$$X \stackrel{def}{=} \sum_{j=1}^{n} a_j X_j$$

where $n \in N$, $a_j \in Act$ and $X_j \in Var(\Delta)$.

It is easy to see that a process is regular iff it can reach only finitely many states up to bisimilarity. In [Mil89] it is shown, that regular processes can be represented in the normal form just defined. Thus a process Δ is regular iff there is a regular process Δ' in normal form such that $\Delta \sim \Delta'$. Now we present several propositions which concern regularity of normed BPA_τ and normed BPP_τ processes. Proofs can be found in [Kuč95].

Proposition 1. *Let Δ be a normed BPA_τ or BPP_τ process. The problem whether Δ is regular is decidable in polynomial time. Moreover, if Δ is regular then a regular process Δ' in normal form such that $\Delta \sim \Delta'$ can be effectively constructed.*

Definition 4. *Let Δ be a normed BPA_τ (or BPP_τ) process. A variable $Y \in Var(\Delta)$ is* growing *if $Y \to^* Y.\alpha$ (or $Y \to^* Y|\alpha$) where $\alpha \in Var(\Delta)^*$ such that $Length(\alpha) \geq 1$ (or $\alpha \in Var(\Delta)^\otimes$ such that $card(\alpha) \geq 1$).*

Proposition 2. *A normed BPA_τ (or BPP_τ) process Δ is non-regular iff $Var(\Delta)$ contains a growing variable Y such that there is a reachable state of the form $Y.\beta$ where $\beta \in Var(\Delta)^*$ (or the state Y is reachable).*

Remark 3. *If Δ is a normed BPA_τ (or BPP_τ) process and α is a BPA_τ (or BPP_τ) expression whose variables are defined in Δ, then Proposition 2 can be applied also to α — each such expression denotes a process in the sense of Remark 1. Namely variables of Δ are BPA_τ and BPP_τ expressions—hence we can also speak about* regular variables.

3 The characterisation of $\mathbf{nBPA_\tau \cap nBPP_\tau}$

In this section we give an exact characterisation of those normed processes which can be equivalently defined by both BPA_τ and BPP_τ syntax.

Definition 5 (the class $\mathbf{nBPA_\tau \cap nBPP_\tau}$). *Let $nBPA_\tau$ and $nBPP_\tau$ denote the classes of normed BPA_τ and normed BPP_τ processes, respectively. We define the class $nBPA_\tau \cap nBPP_\tau$ in the following way:*

$$\begin{aligned} nBPA_\tau \cap nBPP_\tau \;=\; & \{\Delta \in nBPA_\tau, \;\mid\; \exists \Delta' \in nBPP_\tau \;\; \textit{such that } \Delta \sim \Delta'\} \cup \\ & \{\Delta \in nBPP_\tau, \;\mid\; \exists \Delta' \in nBPA_\tau \;\; \textit{such that } \Delta \sim \Delta'\} \end{aligned}$$

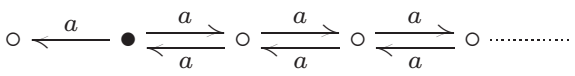
The class $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ can be seen as a "semantical intersection" of nBPA_τ and nBPP_τ. It is clearly nonempty because each normed finite-state process belongs to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$. But $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ contains also processes with infinitely many states—assume the following process:

$$X \;\stackrel{def}{=}\; a(X|X) + a \tag{1}$$

X is a normed BPP_τ process with infinitely many states. If we replace the operator '|' with '.', we obtain a bisimilar BPA_τ process:

$$\overline{X} \;\stackrel{def}{=}\; a(\overline{X}.\overline{X}) + a \tag{2}$$

Clearly $X \sim \overline{X}$ because transition systems generated by those processes are even isomorphic:



Now we modify the process X slightly:

$$X \stackrel{def}{=} a(X|X) + a + \overline{a} \tag{3}$$

Although the process (3) does not differ from the process (1) too much, it is not hard to prove that there is *no* BPA_τ process bisimilar to (3).

Now we prove that each normed BPP_τ processes from $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ can be represented in a special normal form, denoted $\mathrm{INF}_{\mathrm{BPP}}$ (**I**ntersection **N**ormal **F**orm for normed BPP_τ processes). Before the definition of $\mathrm{INF}_{\mathrm{BPP}}$ we first introduce the notion of *reduced* processes:

Definition 6 (reduced processes). *Let Δ be a normed BPA_τ (or BPP_τ) process in GNF. We say that Δ is* reduced *if*

1. *variables of $Var(\Delta)$ are pairwise non-bisimilar*
2. *for each $V \in Var(\Delta)$ there is a reachable state of the form $V.\alpha$, where $\alpha \in Var(\Delta)^*$ (or the state V is reachable).*

As bisimilarity is decidable for normed BPA_τ and normed BPP_τ processes in polynomial time (see [HJM94a], [HJM94b]), the first condition can be assumed w.l.o.g. Variables which do not fulfil the second condition cannot contribute to the behaviour of Δ and they can be effectively removed in polynomial time. Hence we can assume (w.l.o.g.) that a normed BPA_τ or BPP_τ process Δ is reduced.

Definition 7 ($\mathbf{INF_{BPP}}$). *Let Δ be a normed reduced BPP_τ process in GNF.*

1. *A variable $Z \in Var(\Delta)$ is* simple *if all summands in the def. equation for Z are of the form aZ^i, where $a \in Act$ and $i \in N \cup \{0\}$. Moreover, at least one of those summands must be of the form aZ^k where $a \in Act$ and $k \geq 2$. Finally, the def. equation for Z must not contain two summands of the form $b, \overline{b}$, where $b \in Act$.*
2. *The process Δ is said to be in INF_{BPP} if whenever $a\alpha$ is a summand in a def. equation from Δ such that $Length(\alpha) \geq 2$, then $\alpha = Z^i$ for some simple variable Z and $i \geq 2$.*

Note that if Z is a simple variable, then $|Z| = 1$ because Z could not be normed otherwise.

Example 1. *The following process as well as process (1) are in INF_{BPP}, while the processes (3) is not:*

$$\begin{aligned} X &\stackrel{def}{=} aY + b(Z|Z) + b + \overline{b} \\ Y &\stackrel{def}{=} cY + bX + a(Z|Z|Z) \\ Z &\stackrel{def}{=} a(Z|Z) + \overline{a}(Z|Z|Z) + b + \overline{a} \end{aligned}$$

Remark 4. *The set of all reachable states of a process Δ in INF_{BPP} looks as follows:*

$$Var(\Delta) \cup \{Z^i \mid Z \in Var(\Delta) \text{ is a simple variable and } i \in N \cup \{0\}\}$$

Proposition 3. *Each process Δ in INF_{BPP} belongs to $nBPA_\tau \cap nBPP_\tau$.*

Proof: We show how to construct a normed BPA_τ process $\overline{\Delta}$, which is bisimilar to Δ. First we need to define the notion of *closed* simple variables—a simple variable $Z \in Var(\Delta)$ is closed if the following condition holds: If the def. equation for Z contains two summands of the form $bZ^i, \overline{b}Z^j$, then it also contains a summand τZ^{i+j-1} (note that Z is simple, hence the case $i = j = 0$ is impossible).

The set of variables of $\overline{\Delta}$ looks as follows: for each $V \in Var(\Delta)$ we fix a fresh variable $\overline{V}$. Moreover, for each simple non-closed variable $Z \in Var(\Delta)$ we also fix a fresh variable $\overline{Z}_C$. Now we can start to transform Δ into $\overline{\Delta}$. For each equation $Y \stackrel{def}{=} \sum_{i=1}^{n} a_i\alpha_i$ of Δ we add the equation $\overline{Y} \stackrel{def}{=} \sum_{i=1}^{n} a_i\mathcal{T}(\alpha_i)$ to $\overline{\Delta}$, where $\mathcal{T}$ is defined as follows:

1. $\mathcal{T}(V) = \overline{V}$, where $V \in Var(\Delta)$.
2. $\mathcal{T}(Z^i) = \overline{Z}^i$, where $i \geq 2$ and $Z \in Var(\Delta)$ is a closed simple variable.
3. $\mathcal{T}(Z^i) = \overline{Z}_C{}^{i-1}.\overline{Z}$, where $i \geq 2$ and $Z \in Var(\Delta)$ is a non-closed simple variable.

The defining equation for $\overline{Z}_C$, where $Z \in Var(\Delta)$ is a non-closed simple variable, is constructed using following rules:

1. if aZ^i is a summand in the def. equation for Z, then $a\overline{Z}_C{}^i$ is a summand in the def. equation for $\overline{Z}_C$ in $\overline{\Delta}$.
2. if bZ^i, $\overline{b}Z^j$ are summands in the def. equation for Z, then $\tau\overline{Z}_C{}^{i+j-1}$ is a summand in the def. equation for $\overline{Z}_C$ in $\overline{\Delta}$.

The fact $\Delta \sim \overline{\Delta}$ is easy to check. $\square$

Example 2. *If we apply the transformation algorithm to the process from Example 1, we obtain the following bisimilar BPA_τ process:*

$$\begin{aligned}
\overline{X} &\stackrel{def}{=} a\overline{Y} + b(\overline{Z}_C.\overline{Z}) + b + \overline{b} \\
\overline{Y} &\stackrel{def}{=} c\overline{Y} + b\overline{X} + a(\overline{Z}_C.\overline{Z}_C.\overline{Z}) \\
\overline{Z} &\stackrel{def}{=} a(\overline{Z}_C.\overline{Z}) + \overline{a}(\overline{Z}_C.\overline{Z}_C.\overline{Z}) + b + \overline{a} \\
\overline{Z}_C &\stackrel{def}{=} a(\overline{Z}_C.\overline{Z}_C) + \overline{a}(\overline{Z}_C.\overline{Z}_C.\overline{Z}_C) + b + \overline{a} + \tau(\overline{Z}_C.\overline{Z}_C.\overline{Z}_C.\overline{Z}_C) + \tau\overline{Z}_C
\end{aligned}$$

Now we prove that each normed BPP_τ process from $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ is bisimilar to a process in $\mathrm{INF}_{\mathrm{BPP}}$. Several auxiliary definitions and lemmas are needed:

Definition 8 (Assoc sets). *Let Δ be a normed reduced BPP_τ process in 3-GNF. For each growing variable $Y \in Var(\Delta)$ we define the set $Assoc(Y) \subseteq Var(\Delta)$ in the following way:*

$$\begin{aligned}
Assoc(Y) \;=\; & \{P \in Var(\Delta),\ Y \to^* P\} \cup \\
& \{P \in Var(\Delta),\ P|Y \text{ is a reachable state of } \Delta\}
\end{aligned}$$

A variable $L \in Var(\Delta)$ is lonely *if $L \notin Assoc(Y)$ for any growing variable $Y \in Var(\Delta)$.*

Lemma 1. *Let Δ be a normed reduced BPP_τ process in 3-GNF such that $\Delta \in nBPA_\tau \cap nBPP_\tau$. Let $Y \in Var(\Delta)$ be a growing variable. Then there is exactly one variable $Z_Y \in Var(\Delta)$ such that:*

- *Z_Y is non-regular and $|Z_Y| = 1$*
- *if $P \in Assoc(Y)$ then $P \sim Z_Y^{|P|}$ and Z_Y is reachable from P*
- *if $a\alpha$ is a summand in the defining equation for Z_Y in Δ, then $\alpha \sim Z_Y^{|\alpha|}$*

Proof: As Y is growing, $Y \to^* Y|\beta$ where $\beta \in Var(\Delta)^\otimes$, $\beta \neq \emptyset$. As Δ is normed and in GNF, there is $Z_Y \in Var(\Delta)$, $|Z_Y| = 1$ such that $\beta \to^* Z_Y$. Hence $Y \to^* Y|\beta^i \to^* Y|Z_Y^i$ for any $i \in N$ (note that Z_Y is reachable from Y). From this and the definition of *Assoc* set we can easily conclude that if $P \in Assoc(Y)$ then the state $P|Z_Y^i$ is reachable for any $i \in N$.

As $\Delta \in \mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, there is a normed BPA_τ process Δ' in GNF such that $\Delta \sim \Delta'$. Let $n = |P|$, $m = \max\{|A|,\ A \in Var(\Delta')\}$. The

state $P|Z_Y^{n.m}$ is a reachable state of Δ and therefore there is $\gamma \in Var(\Delta')^*$ such that $P|Z_Y^{n.m} \sim \gamma$. Bisimilar states must have the same norm, hence γ is a sequence of at least $n+1$ variables — $\gamma = A_1.A_2 \ldots A_{n+1}.\delta$ where $\delta \in Var(\Delta')^*$. As $|P| = n$, there is $s \in Act^*$, $Length(s) = n$ such that $P \stackrel{s}{\to}{}^* \epsilon$ — hence $P|Z_Y^{n.m} \stackrel{s}{\to}{}^* Z_Y^{n.m}$. The state $A_1.A_2 \ldots A_{n+1}.\delta$ must be able to match the norm reducing sequence of actions s. As $Length(s) = n$, at most the first n variables of $A_1.A_2 \ldots A_{n+1}.\delta$ can contribute to the sequence s, i.e. $A_1.A_2 \ldots A_{n+1}.\delta \stackrel{s}{\to}{}^* \eta.A_{n+1}.\delta$ where $\eta \in Var(\Delta')^*$. As Δ' is normed, there is $t \in Act^*$, $Length(t) = |\eta|$ such that $\eta.A_{n+1}.\delta \stackrel{t}{\to}{}^* A_{n+1}.\delta$. The state $Z_Y^{n.m}$ can match the norm reducing sequence t only by removing $Length(t)$ copies of Z_Y:

$$\begin{array}{ccc} P|Z_Y^{n.m} & \sim & A_1 \ldots A_{n+1}.\delta \\ \downarrow s & & \downarrow s \\ Z_Y^{n.m} & \sim & \eta.A_{n+1}.\delta \\ \downarrow t & & \downarrow t \\ Z_Y^{n.m-|\eta|} & \sim & A_{n+1}.\delta \end{array}$$

Now let $k = Length(s)+Length(t)$ (i.e. $k = |A_1 \ldots A_n|$). Clearly $k \leq n.m$. As $|Z_Y| = 1$, there is $p \in Act^*$, $Length(p) = k$ such that $P|Z_Y^{n.m} \stackrel{p}{\to}{}^* P|Z_Y^{n.m-k}$. The norm reducing sequence p must be matched by $A_1.A_2 \ldots A_{n+1}.\delta$. As $Length(p) = k = |A_1 \ldots A_n|$, we have $A_1.A_2 \ldots A_{n+1}.\delta \stackrel{p}{\to}{}^* A_{n+1}.\delta$ and $P|Z_Y^{n.m-k} \sim A_{n+1}.\delta$. By transitivity of $\sim$ we now obtain $P|Z_Y^{n.m-k} \sim Z_Y^{n.m-|\eta|}$, hence $P \sim Z_Y^{|P|}$.

As the variable Y is non-regular and $Y \sim Z_Y^{|Y|}$, the variable Z_Y is also non-regular. Moreover, Z_Y is a unique variable with the property $P \sim Z_Y^{|P|}$ for each $P \in Assoc(Y)$ because Δ is reduced.

A similar argument can be used to prove that Z_Y is reachable from each $P \in Assoc(Y)$. As P is normed, $P \to^* P'$ where $|P'| = 1$. As $P \sim Z_Y^{|P|}$, $P' \sim Z_Y$ and hence $P' = Z_Y$.

It remains to check that if $a\alpha$ is a summand of the defining equation for Z_Y in Δ then $\alpha \sim Z_Y^{|\alpha|}$. But each variable $V \in \alpha$ belongs to $Assoc(Y)$ (because $Y \to^* Z_Y \to^* V$) and thus $V \sim Z_Y^{|V|}$. Hence $\alpha \sim Z_Y^{|\alpha|}$. □

Remark 5. *In the rest of this paper the symbol Z_Y, where $Y \in Var(\Delta)$ is a growing variable, always denotes the unique variable of Lemma 1.*

Lemma 2. *Let Δ be a normed reduced BPP_τ process in 3-GNF such that $\Delta \in nBPA_\tau \cap nBPP_\tau$. Let $A|B$ be a reachable state of Δ such that $A \in$*

$Assoc(Y)$ and $B \in Assoc(Q)$ for some growing variables $Y, Q \in Var(\Delta)$. Then $Z_Y = Z_Q$.

Proof: As Δ is reduced, it suffices to prove that $Z_Y \sim Z_Q$. As $A \in Assoc(Y)$, $A \to^* Z_Y$ (see Lemma 1). Similarly, $B \to^* Z_Q$ and hence $Z_Y|Z_Q$ is a reachable state of Δ. As Z_Q is non-regular, it can reach a state of an arbitrary norm—for each $i \in N$ there is $\alpha_i \in Var(\Delta)^{\otimes}$ such that $Z_Q \to^* \alpha_i$ and $|\alpha_i| = i$. Clearly $\alpha_i \sim Z_Q^i$ because each variable from α_i belongs to $Assoc(Q)$. Hence $Z_Y|\alpha_i \sim Z_Y|Z_Q^i$.

As $\Delta \in \mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, there is a normed BPA_τ process Δ' in GNF such that $\Delta \sim \Delta'$. Let $m = \max\{|V|,\ V \in Var(\Delta')\}$. The state $Z_Y|\alpha_m$ is a reachable state of Δ and therefore there is $\gamma \in Var(\Delta')^*$ such that $Z_Y|\alpha_m \sim \gamma$ and hence also $Z_Y|Z_Q^m \sim \gamma$. Moreover, γ is a sequence of at least two variables.

Now we can use a similar construction as in the proof of Lemma 1 and conclude that $Z_Y|Z_Q^j \sim Z_Q^{j+1}$ for some $j \in N$. This implies $Z_Y \sim Z_Q$. □

Lemma 3. *Let Δ be a normed reduced BPP_τ process in 3-GNF such that $\Delta \in nBPA_\tau \cap nBPP_\tau$. Let $L|A$ be a reachable state of Δ such that L is a lonely variable. Then A is a regular process.*

Proof: Let us assume that A is not regular. Then $A \to^* Y$, where $Y \in Var(\Delta)$ is a growing variable (see Proposition 2). But then $L|A \to^* L|Y$, thus $L \in Assoc(Y)$—we have a contradiction. □

Proposition 4. *Let Δ be a normed BPP_τ process from $nBPA_\tau \cap nBPP_\tau$. Then there is a process Δ' in INF_{BPP} such that $\Delta \sim \Delta'$.*

Proof: We can assume (w.l.o.g.) that Δ is reduced and in 3-GNF. The process Δ' can be obtained by the following transformation of defining equations of Δ (which can also add completely new variables and equations): if $X \stackrel{def}{=} \sum_{j=1}^m a_j\alpha_j$ is a defining equation from Δ, then $X \stackrel{def}{=} \sum_{j=1}^m \mathcal{T}(a_j\alpha_j)$ is added to Δ', where $\mathcal{T}$ is defined as follows:

- if $card(\alpha_j) \leq 1$ then $\mathcal{T}(a_j\alpha_j) = a_j\alpha_j$
- if $card(\alpha_j) = 2$ (i.e. $\alpha_j = A|B$) then there are three possibilities:
  1. $A \in Assoc(Y)$ and $B \in Assoc(Q)$ for some growing variables $Y, Q \in Var(\Delta)$. Then $A \sim Z_Y^{|A|}$ and $B \sim Z_Q^{|B|}$ (see Lemma 1). As $A|B$ is a reachable state, we can conclude (with a help of Lemma 2) that $Z_Y = Z_Q$, hence $A|B \sim Z_Y^{|A|+|B|}$. Thus $\mathcal{T}(a(A|B)) = a(Z_Y^{|A|+|B|})$.

2. $A \in Assoc(Y)$ for some growing variable $Y \in Var(\Delta)$ and B is lonely. But then $A \sim Z_Y^{|A|}$ and as Z_Y is not regular, A is not regular as well. As the state $A|B$ is reachable and B is lonely, it contradicts Lemma 3. Hence this case is in fact impossible (as well as the case when A is lonely and $B \in Assoc(Q)$).
3. A and B are lonely. Then A and B are regular (due to Lemma 3) and therefore the state $A|B$ is also regular. Each regular process can be represented in normal form (see Definition 3). Let $\Delta_{A|B}$ be a regular process in normal form which is bisimilar to $A|B$. We can assume (w.l.o.g.) that $Var(\Delta_{A|B}) \cap Var(\Delta') = \emptyset$. $\mathcal{T}$ adds all equations from $\Delta_{A|B}$ to Δ' and $\mathcal{T}(a(A|B)) = a.N$ where N is the leading variable of $\Delta_{A|B}$.

The transformation $\mathcal{T}$ preserves bisimilarity—hence $\Delta \sim \Delta'$. It remains to check that Δ' is in $\mathrm{INF}_{\mathrm{BPP}}$. Clearly each summand of each defining equation from Δ' is of the form which is admitted by $\mathrm{INF}_{\mathrm{BPP}}$. If aZ^j is a summand of a defining equation in Δ' such that $j \geq 2$, then $Z = Z_Y$ for some growing variable $Y \in Var(\Delta)$. Let $a\alpha$ be a summand in the original defining equation for Z_Y in Δ. We need to show that each such summand must have been tranformed into $aZ_Y^{|\alpha|}$ by $\mathcal{T}$. But it is obvious as each variable from α belongs to $Assoc(Y)$. If α is composed of a single variable V, then $V = Z_Y$ because $V \sim Z_Y$ (due to Lemma 1) and Δ is reduced. Moreover, at least one summand in the defining equation for Z_Y in Δ' is of the form aZ_Y^l where $l \geq 2$, because Z_Y would be regular otherwise. To complete the proof we need to show that the defining equation for Z_Y in Δ' cannot contain two summands of the form $b, \overline{b}$. Assume the converse. As $\Delta' \in \mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, there is a normed BPA_τ process Δ_2 in GNF such that $\Delta' \sim \Delta_2$. As Z_Y^i is a reachable state of Δ' for each $i \in N \cup \{0\}$ (see Remark 4), there is $\alpha_i \in Var(\Delta_2)^*$ such that $Z_Y^i \sim \alpha_i$ for each i. Moreover, we can assume (w.l.o.g.) that each α_i is of maximal length, i.e. if $\alpha_i \sim \beta$ for some $\beta \in Var(\Delta_2)^*$, then $Length(\alpha_i) \geq Length(\beta)$. Let k be the minimal number with the property $Length(\alpha_k) \geq 2$. Clearly $Length(\alpha_k) = 2$, because otherwise we could easily obtain a contradiction with the minimality of k. Hence $\alpha_k = P.Q$ for some $P, Q \in Var(\Delta_2)$. As $Z_Y^k \xrightarrow{b} Z_Y^{k-1}$, we also have $P.Q \xrightarrow{b} \gamma$ for some $\gamma \sim \alpha_{k-1}$. By the definition of α_i and k, γ must be composed of a single variable. The only such state which can be entered by $P.Q$ in one step is Q, hence $\alpha_{k-1} \sim Q$. As the defining equation for Z_Y contains two summands $b, \overline{b}$, we also have a transition $Z_Y^k \xrightarrow{\tau} Z_Y^{k-2}$. But $P.Q$ cannot reach a state which is bisimilar to α_{k-2} in one step, because α_{k-2} is (again by the definition of α_i and k) composed of at most one variable which must be different from Q

because $\alpha_{k-1} \not\sim \alpha_{k-2}$. Hence $\alpha_k \not\sim Z_Y^k$ and we have a contradiction. $\square$

Propositions 3 and 4 give us the classification of $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ in terms of BPP_τ syntax:

Theorem 1. *The class $nBPA_\tau \cap nBPP_\tau$ contains exactly (up to bisimilarity) normed BPP_τ processes in INF_{BPP}.*

The class $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ can also be characterised using BPA_τ syntax. To do this, we introduce a special normal form for normed BPA_τ processes:

Definition 9 ($\mathbf{INF_{BPA}}$). *Let Δ be a normed reduced BPA_τ process in GNF.*

1. *Let $X, Y \in Var(\Delta)$ be non-regular variables. We say that Y is a* communication closure *(C-closure) of X if the following conditions hold:*

   - *All summands in the def. equation for X are either of the form a where $a \in Act$, or $a(Y^i.X)$ where $a \in Act$ and $i \in N \cup \{0\}$. Moreover, at least one summand is of the form $a(Y^k.X)$ where $k \geq 1$.*
   - *All summands in the def. equation for Y are of the form aY^i, where $a \in Act$ and $i \in N \cup \{0\}$.*
   - *aY^i is a summand in the def. equation for Y iff one of the following conditions holds:*
     (a) *$i = 0$ and a is a summand in the def. equation for X.*
     (b) *$i \geq 1$ and $a(Y^{i-1}.X)$ is a summand in the def. equation for X.*
     (c) *$a = \tau$ and there are two summands of the form $b\alpha_1, \overline{b}\alpha_2$ in the def. equation for X such that $i = Length(\alpha_1) + Length(\alpha_2) - 1$ (note that this condition ensures that def. equations for X, Y do not contain two summands of the form $b, \overline{b}$).*

2. *The process Δ is said to be in INF_{BPA} if whenever $a\alpha$ is a summand in a def. equation from Δ such that $Length(\alpha) \geq 2$, then $\alpha = X^i.Y$ for some $i \in N$ and $X, Y \in Var(\Delta)$ such that Y is a C-closure of X (note that X, Y need not be different—there can be variables which are C-closed by themselves).*

Note that if Y is a C-closure of X, then $|Y| = |X| = 1$. Another interesting property of X and Y is presented in the following remark.

Remark 6. *It is easy to check that if Y is a C-closure of X, then $Y^i.X \sim \overline{X}^{i+1}$ where $\overline{X}$ is a normed BPP process composed of a single variable whose def. equation is obtained from the def. equation for X by substituting '.' with '|' and replacing each occurence of X and Y with $\overline{X}$.*

Theorem 2. *The class $nBPA_\tau \cap nBPP_\tau$ contains exactly (up to bisimilarity) normed BPA_τ processes in INF_{BPA}.*

Proof: Each BPA_τ process in $\mathrm{INF}_{\mathrm{BPA}}$ belongs to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, as a bisimilar BPP_τ process can be easily constructed by an algorithm which is inverse to the algorithm presented in the proof of Proposition 3 (see Remark 6). The fact that for each normed BPA_τ process in $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ there is a bisimilar BPA_τ process in $\mathrm{INF}_{\mathrm{BPA}}$ follows directly from Proposition 3 and Proposition 4 (note that the algorithm presented in the proof of Proposition 3 returns a BPA_τ process which is almost in $\mathrm{INF}_{\mathrm{BPA}}$—the only "problem" is that it can contain different bisimilar variables and hence it need not be reduced.). □

BPA_τ and BPP_τ processes are often defined in a simplified form, where 'τ' cannot be used as an action prefix and pure merge operator '$\|$', which does not allow synchronisations on dual actions, is used instead of '|'. These process classes are denoted BPA and BPP by convention. So far we have investigated the intersection of nBPA_τ and nBPP_τ processes. It was desirable to work with this unrestricted syntax, because we could also examine when it is possible to simulate "real" communications in a BPP_τ process by a sequential BPA_τ process. Obviously these results applies to nBPA/nBPP processes as well. But the characterisation of $\mathrm{nBPA} \cap \mathrm{nBPP}$ is much simpler and therefore we present it explicitly.

Definition 10 (INF). *Let Δ be a normed reduced BPA or BPP process in GNF.*

1. *A variable $Z \in Var(\Delta)$ is* simple *if all summands in the def. equation for Z are of the form aZ^i, where $a \in Act$ and $i \in N \cup \{0\}$. Moreover, at least one of those summands must be of the form aZ^k where $a \in Act$ and $k \geq 2$.*

2. *The process Δ is said to be in INF if whenever $a\alpha$ is a summand in a def. equation from Δ such that $Length(\alpha) \geq 2$ (or $card(\alpha) \geq 2$), then $\alpha = Z^i$ for some simple variable Z and $i \geq 2$.*

Note that normed BPA (or BPP) processes in INF have a nice property—a bisimilar normed BPP (or BPA) process can be obtained just by replacing '.' operator with '$\|$' operator (or by replacing '$\|$' operator with '.' operator).

Theorem 3. *The class nBPA ∩ nBPP contains exactly (up to bisimilarity) normed BPA (or BPP) processes in INF.*

4 Deciding whether $\Delta \in \mathbf{nBPA}_\tau \cap \mathbf{nBPP}_\tau$

In this section we prove that the problem whether a given normed BPA_τ or BPP_τ process Δ belongs to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ is decidable in polynomial time. The technique is essentially similar in both cases—we check if each summand of each defining equation of Δ whose form is not admitted by $\mathrm{INF}_{\mathrm{BPA}}$ or $\mathrm{INF}_{\mathrm{BPP}}$ can be in principal transformed so that requirements of $\mathrm{INF}_{\mathrm{BPA}}$ or $\mathrm{INF}_{\mathrm{BPP}}$ are fulfilled. We also present simplified versions of our algorithms which work for normed BPA and BPP processes.

Next we show how to modify presented algorithms so that they become constructive. Unfortunately, these algorithms are no longer polynomial. We start with some definitions:

Definition 11 ($\mathbf{S}(\boldsymbol{\Delta}), \mathbf{R}(\boldsymbol{\Delta})$ and $\mathbf{G}(\boldsymbol{\Delta})$ sets). *Let Δ be a normed BPA_τ or BPP_τ process in GNF.*

- *the set $S(\Delta) \subseteq Var(\Delta)$ is composed of all variables V such that $|V| = 1$, V is non-regular and if $a\alpha$ is a summand in the defining equation for V in Δ, then $\alpha \sim V^{|\alpha|}$.*
- *the set $R(\Delta) \subseteq Var(\Delta)$ contains all regular variables of Δ.*
- *the set $G(\Delta) \subseteq Var(\Delta)$ contains all growing variables of Δ.*

The sets $S(\Delta)$, $R(\Delta)$ and $G(\Delta)$ can be constructed in polynomial time because bisimilarity and regularity are decidable in polynomial time for normed BPA_τ and normed BPP_τ processes (see [HJM94a], [HJM94b] and [Kuč95]).

If Δ is a normed BPA_τ (or BPP_τ) process from $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, then there is Δ' in $\mathrm{INF}_{\mathrm{BPA}}$ (or $\mathrm{INF}_{\mathrm{BPP}}$) such that $\Delta \sim \Delta'$. In case of normed BPP_τ processes the set $S(\Delta)$ contains in fact variables which can be (potentially) bisimilar to simple variables of Δ'. In case of normed BPA_τ processes the set $S(\Delta)$ contains variables which can be bisimilar to C-closures of variables from $Var(\Delta')$.

Now we present three lemmas which help us to prove the corectness of the algorithm below (it decides whether a given normed BPP_τ process belongs to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$).

Lemma 4. *Let Δ be a normed reduced BPP_τ process in 3-GNF and let $a(A|B)$ be a summand of a defining equation from Δ such that A is regular and B is non-regular. Then $\Delta \notin nBPA_\tau \cap nBPP_\tau$.*

Proof: Assume there is a normed BPP_τ process Δ' in $\mathrm{INF}_{\mathrm{BPP}}$ such that $\Delta \sim \Delta'$. Let $n = \max\{|Y|,\ Y \in \mathit{Var}(\Delta')\}$. As B is non-regular, it can reach a state of an arbitrary norm—let $B \to^* \beta$ where $|\beta| > n$. Then $A|\beta$ is a reachable state of Δ and thus $A|\beta \sim \beta'$ for some reachable state β' of Δ'. As $|\,A|\beta\,| > n$, we can conclude that $\beta' \equiv Z^{|\,A|\beta\,|}$ where $Z \in \mathit{Var}(\Delta')$ is a simple variable (see Remark 4). Hence $A \sim Z^{|A|}$ and as each simple variable is growing (see Definition 7), it contradicts regularity of A. □

Lemma 5. *Let Δ be a normed reduced BPP_τ process in 3-GNF such that $\Delta \in nBPA_\tau \cap nBPP_\tau$. Let $a(A|B)$ be a summand of a defining equation from Δ such that A and B are non-regular. Then there is exactly one variable $Z \in S(\Delta)$ such that $A|B \sim Z^{|\,A|B\,|}$.*

Proof: Let Δ' be a normed BPP_τ process in $\mathrm{INF}_{\mathrm{BPP}}$ such that $\Delta \sim \Delta'$. Let $n = \max\{|Y|,\ Y \in \mathit{Var}(\Delta')\}$. Using the same argument as in the proof of Lemma 4 we obtain $A \sim P^{|A|}$, $B \sim Q^{|B|}$ where $P, Q \in \mathit{Var}(\Delta')$ are simple variables. We show that $P \equiv Q$. Let $A \to^* \alpha$ where $|\alpha| > n$. Then clearly $\alpha \sim P^{|\alpha|}$ and as $\alpha|B$ is a reachable state of Δ, $\alpha|B \sim R^{|\alpha|B|}$ where $R \in \mathit{Var}(\Delta')$ is a simple variable. To sum up, we have $\alpha|B \sim P^{|\alpha|}|Q^{|B|} \sim R^{|\,\alpha|B\,|}$. Hence $P \sim R \sim Q$ and thus $P \equiv R \equiv Q$ because Δ' is reduced. As e.g. P is a reachable state of Δ', there is a reachable state γ of Δ such that $P \sim \gamma$. As $|P| = 1$, we can conclude $\gamma \equiv Z$ for some $Z \in \mathit{Var}(\Delta)$ which clearly belongs to $S(\Delta)$. Moreover, Z is unique because Δ is reduced. □

Lemma 6. *Let Δ be a normed BPP_τ process in GNF and let $X \in S(\Delta)$. If the defining equation for X contains two summands of the form $b, \overline{b}$, then $\Delta \notin nBPA_\tau \cap nBPP_\tau$.*

Proof: Assume there is a normed BPP_τ process Δ' in $\mathrm{INF}_{\mathrm{BPP}}$ such that $\Delta \sim \Delta'$. Using the same kind of argument as in the proof of Lemma 4 we obtain that $X \sim Z$ for some simple variable $Z \in \mathit{Var}(\Delta')$. As the def. equation for X contains two summands of the form $b, \overline{b}$ and $X \sim Z$, the def. equation for Z must contain those summands too—hence Z is not simple and we have a contradiction. □

The promised (constructive) algorithm for normed BPP_τ processes is presented on Figure 1. Steps which are executed only by the constructive algorithm are shaded—if we omit everything on a grey background, we obtain a non-constructive polynomial algorithm. The abbreviation "NFR(Δ)" stands for the **N**ormal **F**orm of the **R**egular process Δ, which can be effectively constructed (see Proposition 1). We always assume that NFR(Δ) contains fresh variables which are not contained in any other process we are

working with. When the command return is executed, the algorithm *halts* and returns the value which follows immediately after the keyword return.

The constructive algorithm is not polynomial because the construction of NFR is not polynomial—a normed regular BPP process in 3-GNF with n variables can generally reach exponentially many pairwise non-bisimilar states and each of these states requires a special variable.

Our algorithm for normed BPP$_\tau$ processes works for pure normed BPP processes as well. It suffices to replace the '|' operator with '||' operator in our description. As there are no communications in BPP, the notion of dual action is no longer sensible—hence the second step of our algorithm can be removed in case of normed BPP processes.

Now we provide an analogous algorithm for normed BPA$_\tau$ processes. We start with some auxiliary definitions and lemmas.

Definition 12 (CL sets). *Let Δ be a normed BPA$_\tau$ process in GNF. For each $Y \in S(\Delta)$ we define the set $CL(Y)$, composed of all $X \in Var(\Delta)$ which fulfil the following conditions:*

- *If $a\alpha$ is a summand in the def. equation for X such that $Length(\alpha) \geq 1$, then $\alpha \sim Y^{|\alpha|-1}.X$.*
- *The def. equation for Y contains a summand bisimilar to aY^k, $k \in N \cup \{0\}$, iff one of the following conditions holds:*
  1. *$\alpha = \epsilon$ and the def. equation for X contains a summand a*
  2. *$\alpha \neq \epsilon$ and the def. equation for X contains a summand which is bisimilar to $a(Y^{k-1}.X)$.*
  3. *$a = \tau$ and the def. equation for X contains two summands of the form $b\alpha_1$, $\overline{b}\alpha_2$ such that $k = Length(\alpha_1) + Length(\alpha_2) - 1$.*

It is easy to see that the set $CL(Y)$ can be constructed in polynomial time for each $Y \in S(\Delta)$. The following lemma is due to D. Caucal (see [Cau88]):

Lemma 7. *Let Δ, Δ' be normed BPA$_\tau$ processes in GNF and let $\alpha, \beta \in Var(\Delta)$, $\alpha', \beta' \in Var(\Delta')$ such that $\beta \sim \beta'$ and $\alpha.\beta \sim \alpha'.\beta'$. Then $\alpha \sim \alpha'$*

Lemma 8. *Let Δ, Δ' be normed BPA$_\tau$ processes. Let $A_1, \ldots, A_k \in Var(\Delta)$, $X, Y \in Var(\Delta')$ such that $|X| = |Y| = 1$ and $A_1.\cdots.A_k \sim Y^l.X$ where $l = |A_1.\cdots.A_k| - 1$. Then $A_k \sim Y^{|A_k|-1}.X$ and $A_i \sim Y^{|A_i|}$ for $1 \leq i < k$.*

Proof: Clearly $A_k \sim Y^{|A_k|-1}.X$. Hence $A_1.\cdots.A_{k-1} \sim Y^{|A_1.\cdots.A_{k-1}|}$ (due to Lemma 7). The fact $A_i \sim Y^{|A_i|}$ for $1 \leq i < k$ can be proved by induction

Algorithm: A constructive test of the membership to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ for normed BPP_τ processes.

Input: A normed reduced BPP_τ process Δ in 3-GNF.

Output: **YES** and a normed BPP_τ process Δ' in $\mathrm{INF}_{\mathrm{BPP}}$ such that $\Delta \sim \Delta'$ if $\Delta \in \mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$.
NO otherwise.

1. Construct the sets $S(\Delta)$, $R(\Delta)$ and $G(\Delta)$.
2. If there is $X \in S(\Delta)$ whose def. equation contains two summands of the form $b, \overline{b}$ then
   return **NO**;
3. If $G(\Delta) = \emptyset$ then
   $\Delta' :=$NFR(Δ) ;
   return **YES** and Δ' ;
4. $\Delta' := \Delta$;
5. for each summand of the form $a(A|B)$ in defining equations of Δ do
   if $A, B \in R(\Delta)$ then
   Construct NFR$(A|B)$;
   Replace the summand $a(A|B)$ with aN in Δ', where N is the leading variable of NFR$(A|B)$;
   $\Delta' := \Delta' \cup$NFR$(A|B)$;
   if ($A \in R(\Delta)$ and $B \notin R(\Delta)$) or ($A \notin R(\Delta)$ and $B \in R(\Delta)$) then
   return **NO**;
   if $A, B \notin R(\Delta)$ then
   if there exists $Z \in S(\Delta)$ such that $A|B \sim Z^{|A|B|}$
   then Replace the summand $a(A|B)$ with $a(Z^{|A|B|})$ in Δ' ;
   else return **NO**;
6. return **YES** and Δ' ;

Figure 1: An algorithm which (constructively) decides the membership to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ for normed BPP_τ processes.

on k. If $k = 2$ then $A_1 \sim Y^{|A_1|}$ and our lemma holds. If $k > 2$, then clearly $A_{k-1} \sim Y^{|A_{k-1}|}$ and due to Lemma 7 we have $A_1.\cdots.A_{k-2} \sim Y^{|A_1.\cdots.A_{k-2}|}$. Now we can use induction hypothesis and conclude that $A_i \sim Y^{|A_i|}$ for $1 \leq i < (k-2)$. □

Lemma 9. *Let Δ be a normed reduced BPA_τ process in 3-GNF such that $\Delta \in nBPA_\tau \cap nBPP_\tau$. Let $Q.\alpha$ be a reachable state of Δ such that $Q \in G(\Delta)$, $\alpha \neq \epsilon$. Then there are unique variables $Y \in S(\Delta)$, $X \in CL(Y)$ such that $Q.\alpha \sim Y^{|Q.\alpha|-1}.X$.*

Proof: As $\Delta \in \mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, there is a normed BPA_τ process Δ' in $\mathrm{INF}_{\mathrm{BPA}}$ such that $\Delta \sim \Delta'$. Let $n = \max\{|A|,\ A \in Var(\Delta')\}$. As Q is growing, $Q \to^* Q.\gamma$ where $\gamma \neq \epsilon$. Hence the state $Q.\gamma^n.\alpha$ is a reachable state of Δ and therefore there is a reachable state δ of Δ' such that $Q.\gamma^n.\alpha \sim \delta$. As $|Q.\gamma^n.\alpha| > n$, we can conclude $\delta = R^{|Q.\gamma^n.\alpha|-1}.S$, where R is a C-closure of S (see Definition 9). Hence $Q.\gamma^n.\alpha \sim R^{|Q.\gamma^n.\alpha|-1}.S$ and due to Lemma 8 we have $\alpha \sim R^{|\alpha|-1}.S$ and $Q \sim R^{|Q|}$, thus $Q.\alpha \sim R^{|Q.\alpha|-1}.S$. Now it suffices to show that there are $Y \in S(\Delta)$, $X \in CL(Y)$ such that $Y \sim R$ and $X \sim S$. As Δ is normed, $Q \xrightarrow{s}{}^* Y$ where $|Y| = 1$ and s is a norm-decreasing sequence of actions. Then $Q.\alpha \xrightarrow{s}{}^* Y.\alpha$ and as $Q.\alpha \sim R^{|Q.\alpha|-1}.S$, the state $R^{|Q.\alpha|-1}.S$ must be able to match the sequence s and enter a state bisimilar to $Y.\alpha$. As s is norm-decreasing and $|R| = 1$, the only such state is $R^{|Y.\alpha|-1}.S$. Hence $Y.\alpha \sim R^{|Y.\alpha|-1}.S$ and due to Lemma 8 we have $Y \sim R$. The fact $Y \in S(\Delta)$ follows directly from Definition 9. As S is a reachable state of Δ', there is a variable $X \in S(\Delta)$ such that $X \sim S$. Clearly $X \in CL(Y)$ (see Definition 9). Variables X, Y are unique because Δ is reduced. □

It is worth noting that the variables X, Y of the previous lemma need not be different—if a normed BPA_τ process Δ belongs to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$, then each $Y \in S(\Delta)$ belongs to $CL(Y)$.

To prove the corectness of our algorithm for normed BPA_τ processes we need some lemmas about summands:

Lemma 10. *Let Δ be a normed reduced BPA_τ process in 3-GNF and let $a(A.B)$ be a summand of a defining equation from Δ such that A is non-regular and B is regular. Then $\Delta \notin nBPA_\tau \cap nBPP_\tau$.*

Proof: As $a(A.B)$ is a summand of a defining equation from Δ and Δ is normed and reduced, there is a reachable state of the form $A.B.\beta$. As A is non-regular, $A \to^* Q.\alpha$ where $Q \in G(\Delta)$. Hence $Q.\alpha.B.\beta$ is a reachable state of Δ and due to Lemma 9 we have $Q.\alpha.B.\beta \sim Y^{|Q.\alpha.B.\beta|-1}.X$ for some $Y \in S(\Delta)$, $X \in CL(Y)$. With a help of Lemma 8 we obtain $B \sim Y^{|B|}$ or

$B \sim Y^{|B|-1}.X$ (the latter possibility holds if $\beta = \epsilon$). As X, Y are growing, it contradicts regularity of B. □

Lemma 11. *Let Δ be a normed reduced BPA_τ process in 3-GNF. Let $a(A.B)$ be a summand of a defining equation from Δ such that A is regular and B is non-regular. Then it is possible to replace the summand $a(A.B)$ with aN where $N \notin Var(\Delta)$ and to add a finite number of new equations in INF_{BPA} to Δ such that the resulting process Δ_1 is bisimilar to Δ.*

Proof: As A is regular, it is possible to construct $\Delta_A := NFR(A)$ such that $Var(\Delta) \cap Var(\Delta_A) = \emptyset$. Now we modify defining equations of Δ_A slightly—each summand of the form a where $a \in Act$ is replaced with aB. The resulting system of equations is in $\mathrm{INF}_{\mathrm{BPA}}$. If we add the modified system Δ_A to Δ and replace the summand $a(A.B)$ with aN where N is the leading variable of Δ_A, we obtain a process Δ_1 which is clearly bisimilar to Δ. □

Lemma 12. *Let Δ be a normed reduced BPA_τ process in 3-GNF and let $a(A.B)$ be a summand of a defining equation from Δ such that A and B are non-regular. Then*

1. *If $\Delta \in nBPA_\tau \cap nBPP_\tau$ then there are unique variables $Y \in S(\Delta)$, $X \in CL(Y)$ such that $B \sim Y^{|B|-1}.X$*
2. *Let $B \sim Y^{|B|-1}.X$ for some $Y \in S(\Delta)$ and $X \in CL(Y)$. If there is a sequence of transitions $A = A_0 \stackrel{a_0}{\to} A_1.\alpha_1 \stackrel{a_1}{\to} A_2.\alpha_2 \stackrel{a_2}{\to} \cdots \stackrel{a_k}{\to} A_k.\alpha_k$ such that $k \geq 0$, $A_k \in G(\Delta)$ and $A_k.\alpha_k \not\sim Y^{|A_k.\alpha_k|}$, then $\Delta \notin nBPA_\tau \cap nBPP_\tau$.*
3. *Let $B \sim Y^{|B|-1}.X$ for some $Y \in S(\Delta)$ and $X \in CL(Y)$. If for each sequence of transitions $A = A_0 \stackrel{a_0}{\to} A_1.\alpha_1 \stackrel{a_1}{\to} A_2.\alpha_2 \stackrel{a_2}{\to} \cdots \stackrel{a_k}{\to} A_k.\alpha_k$ such that $A_k \in G(\Delta)$ the state $A_k.\alpha_k$ is bisimilar to $Y^{|A_k.\alpha_k|}$, then the summand $a(A.B)$ can be replaced with aN where $N \notin Var(\Delta)$ and a finite number of new equations in INF_{BPA} can be added to Δ such that the resulting process Δ_2 is bisimilar to Δ.*

Proof:
1. As A is non-regular, $A \to^* Q.\alpha$ where $Q \in G(\Delta)$. The proof can be easily completed with a help of Lemma 8 and Lemma 9.
2. This is a consequence of Lemma 8 and Lemma 9.
3. It suffices to realize that if $A = A_0 \stackrel{a_0}{\to} A_1.\alpha_1 \stackrel{a_1}{\to} A_2.\alpha_2 \stackrel{a_2}{\to} \cdots \stackrel{a_k}{\to} A_k.\alpha_k$ is a sequence of transitions such that $A_0, \ldots, A_{k-1} \notin G(\Delta)$ and $A_k \in G(\Delta)$,

then $Length(A_i.\alpha_i) \leq card(Var(\Delta))$ for $0 \leq i \leq k-1$ (here we use the assumption that Δ is in 3-GNF. Naturally, $Length(A_i.\alpha_i)$ is bounded also in case of general GNF). As there are only finitely many sequences of variables of this bounded length, we can introduce a fresh variable for each of them. To construct the process Δ_2, we use a similar procedure as in the proof of Lemma 11. □

An existence of a sequence $A = A_0 \stackrel{a_0}{\to} A_1.\alpha_1 \stackrel{a_1}{\to} A_2.\alpha_2 \stackrel{a_2}{\to} \cdots \stackrel{a_k}{\to} A_k.\alpha_k$ such that $A_k \in G(\Delta)$ and $A_k.\alpha_k \not\sim Y^{|A_k.\alpha_k|}$ is decidable in polynomial time:

Lemma 13. *Let Δ be a normed reduced BPA process in 3-GNF. Let $A \in Var(\Delta)$ be a non-regular variable and let $Y \in S(\Delta)$. The problem whether A can reach a state of the form $Q.\alpha$ where $Q \in G(\Delta)$ and $Q.\alpha \not\sim Y^{|Q.\alpha|}$ is decidable in polynomial time.*

Proof: We divide the set $Var(\Delta)$ into two disjoint subsets of *successful* and *unsuccessful* variables. $P \in Var(\Delta)$ is unsuccessful if one of the following conditions holds:

- P is growing and $P \not\sim Y^{|P|}$.
- The defining equation for P in Δ contains a summand of the form $a(R.S)$ where R is non-regular and $S \not\sim Y^{|S|}$.

A variable is successful if it is not unsuccessful. Furthermore, we define the binary relation '$\Rightarrow$' on $Var(\Delta)$: $U \Rightarrow V$ iff U is successful and the defining equation for U in Δ contains a summand which is of one of the following forms:

- aV
- $a(V.W)$ where $W \in Var(\Delta)$
- $a(W.V)$ where $W \in Var(\Delta)$ is regular

Let '$\Rightarrow^*$' be the reflexive and transitive closure of '$\Rightarrow$'. It is not hard to prove that A can reach a state of the form $Q.\alpha$ where Q is growing and $Q.\alpha \not\sim Y^{|Q.\alpha|}$ iff $A \Rightarrow^* T$ for some unsuccessful variable T. As the relation '$\Rightarrow^*$' can be constructed in polynomial time, the proof is finished. □

An algorithm which decides the membership to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ for normed BPA_τ processes is presented on Figure 2. We use the same notation as in the case of BPP_τ.

Algorithm: A constructive test of the membership to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ for normed BPA_τ processes.

Input: A normed reduced BPA_τ process Δ in 3-GNF.

Output: **YES** and a normed BPA_τ process Δ' in $\mathrm{INF}_{\mathrm{BPA}}$ such that $\Delta \sim \Delta'$ if $\Delta \in \mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$.
NO otherwise.

1. Construct the sets $S(\Delta)$, $R(\Delta)$, $G(\Delta)$ and for each $Y \in S(\Delta)$ construct the set $CL(Y)$.
2. If $(G(\Delta) = \emptyset)$ then
   $\Delta' :=$NFR(Δ) ;
   return **YES** and Δ' ;
3. $\Delta' := \Delta$;
4. for each summand of the form $a(A.B)$ in defining equations of Δ do
   if $A, B \in R(\Delta)$ then
   Construct NFR$(A.B)$;
   Replace the summand $a(A.B)$ with aN in Δ', where N is the leading variable of NFR$(A.B)$;
   $\Delta' := \Delta' \cup$NFR$(A.B)$;
   if $A \notin R(\Delta)$ and $B \in R(\Delta)$ then
   return **NO**;
   if $A \in R(\Delta)$ and $B \notin R(\Delta)$ then
   Construct the process Δ_1 of Lemma 11 ;
   $\Delta' := \Delta_1$;
   if $A, B \notin R(\Delta)$ then
   if there exist $Y \in S(\Delta), X \in CL(Y)$ such that $B \sim Y^{|B|-1}.X$
   then if A can reach a state $Q.\alpha$ where $Q \in G(\Delta)$ and $Q.\alpha \not\sim Y^{|Q.\alpha|}$
   then return **NO**;
   else Construct the process Δ_2 of Lemma 12 ;
   $\Delta' := \Delta_2$;
   else return **NO**;
5. return **YES** and Δ' ;

Figure 2: An algorithm which (constructively) decides the membership to $\mathrm{nBPA}_\tau \cap \mathrm{nBPP}_\tau$ for normed BPA_τ processes.

There is a little shortcoming in the constructive variant of our algorithm for normed BPA_τ processes—as the process Δ_2 of Lemma 12 need not be in 3-GNF, the process Δ' need not remain in 3-GNF either. But each lemma about summands (Lemma 10, 11, 12) is formulated for normed BPA_τ process in 3-GNF. Naturally, it is not a problem to prove analogous lemmas about processes in general GNF—but we think that readability is more important feature than technical accuracy (note there is a similar problem in the constructive variant of our algorithm for normed BPP_τ processes).

In case of normed BPA processes our algorithm must be slightly modified (and simplified). This is a consequence of the fact that a normed BPA process Δ belongs to $\mathrm{nBPA} \cap \mathrm{nBPP}$ iff it can be represented in INF—and INF is a little diffrent from $\mathrm{INF}_{\mathrm{BPA}}$ (see Definitions 10 and 9). Lemma 10 and Lemma 11 are valid also for normed BPA processes. Instead of Lemma 12 we can prove the following (in a similar way):

Lemma 14. *Let Δ be a normed reduced BPA process in 3-GNF and let $a(A.B)$ be a summand of a defining equation from Δ such that A and B are non-regular. Then*

1. *If $\Delta \in nBPA \cap nBPP$ then there is a unique variable $Z \in S(\Delta)$ such that $B \sim Z^{|B|}$*

2. *Let $B \sim Z^{|B|}$ for some $Z \in S(\Delta)$. If there is a sequence of transitions $A = A_0 \stackrel{a_0}{\to} A_1.\alpha_1 \stackrel{a_1}{\to} A_2.\alpha_2 \stackrel{a_2}{\to} \cdots \stackrel{a_k}{\to} A_k.\alpha_k$ such that $k \geq 0$, $A_k \in G(\Delta)$ and $A_k.\alpha_k \not\sim Z^{|A_k.\alpha_k|}$, then $\Delta \notin nBPA \cap nBPP$.*

3. *Let $B \sim Z^{|B|}$ for some $Z \in S(\Delta)$. If for each sequence of transitions $A = A_0 \stackrel{a_0}{\to} A_1.\alpha_1 \stackrel{a_1}{\to} A_2.\alpha_2 \stackrel{a_2}{\to} \cdots \stackrel{a_k}{\to} A_k.\alpha_k$ such that $A_k \in G(\Delta)$ the state $A_k.\alpha_k$ is bisimilar to $Z^{|A_k.\alpha_k|}$, then the summand $a(A.B)$ can be replaced with aN where $N \notin Var(\Delta)$ and a finite number of new equations in INF can be added to Δ such that the resulting process Δ_2 is bisimilar to Δ.*

Our algoritm for normed BPA processes differs from the algorithm on Figure 2 in two things—the sets $CL(Y)$ for $Y \in S(\Delta)$ are not computed at all and the last if statement in the loop of step 4 is replaced with the following code:

```
if A, B ∉ R(Δ) then
    if there exist Z ∈ S(Δ) such that B ∼ Z^|B|
        then if A can reach a state Q.α where Q ∈ G(Δ) and Q.α ≁ Z^|Q.α|
                then return NO;
                else Construct the process Δ2 of Lemma 14 ;
                     Δ' := Δ2 ;
        else return NO;
```

The existence of constructive variants of presented algorithms allow us to prove the following theorem:

Theorem 4. *Bisimilarity is decidable in the union of normed BPA$_\tau$ and normed BPP$_\tau$ processes.*

Proof: Given two normed BPA$_\tau$ or BPP$_\tau$ processes, it is possible to check bisimilarity using algorithms which were published e.g. in [HJM94a] and [HJM94b]. If we get a normed BPP$_\tau$ process Δ_1 and a normed BPA$_\tau$ process Δ_2, then we run one of the constructive algorithms presented earlier. We can choose e.g. the first algorithm with Δ_1 on input. If it answers **NO**, then $\Delta_1 \not\sim \Delta_2$. Otherwise we obtain a normed BPP$_\tau$ process Δ_1' in $\mathrm{INF}_{\mathrm{BPP}}$ which is bisimilar to Δ_1. Now it suffices to check bisimilarity between two normed BPA$_\tau$ processes $\overline{\Delta_1'}$ and Δ_2, where $\overline{\Delta_1'}$ is obtained by running the algorithm presented in the proof of Proposition 3 with Δ_1' on input. □

Note that the corresponding statement holds for normed BPA and BPP processes by specialisation.

5 Conclusions, Related Works and Future Research

We have studied the class nBPA$_\tau$ ∩ nBPP$_\tau$ of those transition systems which can be generated by both nBPA$_\tau$ and nBPP$_\tau$ processes, i.e. the class of normed transition systems which can be equivalently (up to bisimilarity) expressed within the syntax of BPA$_\tau$ and BPP$_\tau$. We have shown that the problems whether a given nBPA$_\tau$ or nBPP$_\tau$ process Δ belongs to nBPA$_\tau$ ∩ nBPP$_\tau$ are decidable in polynomial time. We also provided constructive variants of these algorithms which are unfortunately not polynomial. Hence our algorithms for deciding bisimilarity in the union of nBPA$_\tau$ and nBPP$_\tau$ processes are not polynomial either. A simplified characterisation of nBPA ∩ nBPP and the corresponding algorithms are given as well. An obvious question is, whether there exists a decision algorithm of polynomial complexity. Another problem is, whether our results can be extended to the unnormed cases.

The problem whether a given nBPP process belongs to nBPA $\cap$ nBPP has been independently examined by Blanco in [Bla95] where it is shown that given a nBPP process, one can decide whether it is a nBPA process. His approach is based on well-known specification of properties of BPA transition graphs ([CM90]). A test whether a given nBPP graph fits this specification is given in the work. Consequently, his result does not allow for testing whether a given nBPA process belongs to the intersection as well as for generalisation for '|' operator.

Our result about the classification of nBPA$_\tau$ $\cap$ nBPP$_\tau$ might be of some interest from the point of view of formal languages/automata theory as well. INF (for nBPA processes) can be taken as a special type of CF grammars which generate languages of the form $R.(L_1 \cup \ldots \cup L_n)$, where R is regular and each L_i can be generated by a CF grammar having just one nonterminal and rules of the form $Z \to aZ^k,\ k \geq 0$. Considering language equivalence only, it is obvious that languages of the mentioned type $R.(L_1 \cup \ldots \cup L_n)$ can be recognized by nondeterministic one counter automata. Hence our result on the classification of nBPA $\cap$ nBPP can be considered as a refinement of the result achieved in [Sch92] on the context-freeness of languages generated by Petri nets, as BPP processes form a proper subclass of Petri nets.

We hope a part of our work can be considered as one of the steps towards a solution of the open problem whether bisimilarity is decidable for PA processes. Furthermore, we would like to examine deeper the relationship between classes of behaviours which are generated by different types of syntax (e.g. Petri nets and BPA) and provide similar results like in the case of nBPA$_\tau$ and nBPP$_\tau$ processes—i.e. to characterise the "semantical intersection" and design algorithms which can decide (constructively) the membership to this intersection for both types of syntax. Last but not least corresponding complexity results should be provided.

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